Seminars on Inverse Problems

Theory and Applications

Book of Abstracts Vol. 2

September 20 – December 6, 2022

Foreword

Dear friends,

During the second semester of our seminar series, in between September–December 2022, we happily continued to get together with mathematicians who are passionate about inverse spectral problems. In this brief span of time we were extremely honored to host six speakers from five different countries including Greece, Japan, Malta, Russia, and the United States of America. Our titles varied from *The Marchenko Inversion Method for the Derivative NLS System* to *Inverse Problems for Discrete Operators.*

Just like before, all the information regarding the talks is being collected in this small book with the intention of showing how immensely grateful we are to our speakers for their time and cooperation.

When we first started scheduling this seminar series our main aim was to create an atmosphere where new research ideas could be illustrated. To our surprise and to our delight, we´ve received feedback that our seminars have not only served its purpose as a bridge between researchers, but it also has been useful to young mathematicians as an introduction to the inverse spectral problems course with the help of our YouTube channel.

Starting December 6, we are taking a break until the last week of January. After celebrating the upcoming holiday season we are looking forward to seeing you again in 2023.

We really appreciate all your generous contributions, hard work, and time.

Please stay tuned and to be continued.

Co-organizers (listed in alphabetical order)

F. Ayça Çetinkaya[1], Vladimir Vladičić[2], Biljana Vojvodić[3]

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Abstracts

The Marchenko inversion method for the derivative NLS system

Tuncay Aktosun[1]

[1]University of Texas at Arlington

The Marchenko method is presented for the linear system associated with the derivative NLS (nonlinear Schrödinger) system. The system of linear Marchenko integral equations is derived in order to solve the corresponding inverse scattering problem. Through the use of the inverse scattering transform, solutions are obtained for the derivative NLS system. Explicit solution formulas are developed in closed form by using as input a pair of matrix triplets corresponding to reflectionless scattering data.

This talk is based on the papers and preprints:
[1] Aktosun, T., Ercan, R. **(2019)** Direct and inverse scattering problems for the first-order discrete system associated with the derivative NLS system. *Inverse Probl.* 33, 085002. `https://iopscience.iop.org/article/10.1088/1361-6420/ab2070`
[2] Aktosun, T., Ercan, R. **(2022)** Direct and inverse scattering problems for the first-order discrete system associated with the derivative NLS system. *Stud. Appl. Math.* 148, 270–339. `https://onlinelibrary.wiley.com/doi/full/10.1111/sapm.12441`
[3] Aktosun, T., Ercan, R. **(2022)** The generalized Marchenko method in the inverse scattering problem for a first-order linear system. `https://arxiv.org/abs/2203.02663`
[4] Aktosun, T., Ercan, R., Unlu, M. **(2022)** The Marchenko method to solve the general system of derivative nonlinear Schrödinger equations. `https://arxiv.org/abs/2209.08426`

The recording of the talk can be found at `https://www.youtube.com/watch?v=zDztZlQEd7k`

Inverse spectral problems for classic and modified transmission eigenvalues

Nikolaos Pallikarakis

UC National Technical University of Athens

Research on transmission eigenvalues has been a very active topic in inverse scattering theory. In this talk, we discuss about the inverse transmission eigenvalue problem for the spherically symmetric refractive index. We present some well known uniqueness results for the continuous refractive index as well as for the piecewise continuous case [2]. Next, we highlight the need to introduce modified problems and demonstrate the corresponding modified transmission eigenvalue problem [3]. A new uniqueness result for the inverse problem is derived [4]. We conclude by summarizing similarities and differences among inverse problems using classic and modified transmission eigenvalues.

This talk is based on the papers:

[1] Pallikarakis, N. **(2017)** The inverse spectral problem for the reconstruction of the refractive index from the interior transmission problem. *Ph.D. thesis, National Technical University of Athens.*
[2] Gintides, D., Pallikarakis, N. **(2017)** The inverse transmission eigenvalue problem for a discontinuous refractive index. *Inverse Probl.* 33.https://iopscience.iop.org/article/10.1088/1361-6420/aa5bf0
[3] Gintides, D., Pallikarakis, N., Stratouras, K. **(2021)** On the modified transmission eigen-value problem with an artificial metamaterial background. *Res. Math. Sci. (special issue on transmission eigenvalues)* 8.https://link.springer.com/article/10.1007/s40687-021-00278-z
[4] Gintides, D., Pallikarakis, N., Stratouras, K. **(2022)** Uniqueness of a spherically symmetric refractive index from modified transmission eigenvalues. *Inverse Probl.* 38.https://iopscience.iop.org/article/10.1088/1361-6420/ac7b3f/meta

The recording of the talk can be found at https://www.youtube.com/watch?v=RpBAaKFja4I

Deterministic-Statistical Approach for an Inverse Acoustic Source Problem using Multiple Frequency Limited Aperture Data

Yanfang Liu

George Washington University

We propose a deterministic-statistical method for an inverse source problem using multiple frequency limited aperture far field data. The direct sampling method is used to obtain a disc such that it contains the compact support of the source. The Dirichlet eigenfunctions of the disc are used to expand the source function. Then the inverse problem is recast as a statistical inference problem for the expansion coefficients and the Bayesian inversion is employed to reconstruct the coefficients. The stability of the statistical inverse problem with respect to the measured data is justified in the sense of Hellinger distance. A preconditioned Crank-Nicolson (pCN) Metropolis-Hastings (MH) algorithm is implemented to explore the posterior density function of the unknowns. Numerical examples show that the proposed method is effective for both smooth and non-smooth sources given limited-aperture data.

The recording of the talk can be found at `https://www.youtube.com/watch?v=rK91B8eleos`

A New Numerical Approach for Identifying Source Function in a Plate Equation

Onur Baysal

University of Malta

Kirchoff Plate model is an integral part of most engineering fields including plate and shell structures [1]. In [2], some important identification problems are stated and some properties are analyzed such as stability and uniqueness. In this work we study the inverse problem of identifying the unknown load distribution $f(x,y)$ in the rectangular domain $\Omega := (0,k) \times (0,l)$ such that

$$\begin{cases} u_{tt} + D\Delta^2 u = g(t)f(x,y) \ \texttt{in} \ \Omega_t := \Omega \times (0,T), \\ u(x,y,0) = 0, \ u_t(x,y,0) = 0, \ \texttt{for} \ (x,y) \in \Omega, \\ u = 0, \partial_n = 0, \ \texttt{on} \ \overline{\partial\Omega_i} \times [0,T], \ \texttt{for} \ i = 1,2,3,4, \\ u = 0, -D(vu_{xx} + u_{yy}) = 0, \ \texttt{on} \ \partial\Omega_1 \times [0,T]. \end{cases}$$

Here $u(x,y,t)$ or $(u(x,y,t;f))$ is the displacement at a point $(x,y) \in \overline{\Omega}$ and a time $t \in [0,T]$, $\partial_n u$ denotes the normal derivative of u, $g \in L^2(0,T)$ is the (known) temporal load, $D := E/(1-v^2)$ is the bending stiffness, $v \in (0,1)$ is the Poisson's ratio, E is the elasticity modulus and $\partial\Omega = \sum_{i=1}^4 \partial\Omega_i$ where

$$\partial\Omega_1 = (0,k) \times \{0\}, \ \partial\Omega_2 = \{k\} \times (0,l), \ \partial\Omega_3 = (0,k) \times \{l\}, \ \partial\Omega_4 = \{0\} \times (0,l).$$

In determination of f we have the following boundary observation on $\overline{\Omega_1}$:

$$\theta(x,t) := u_y(x,0,t) \ \texttt{for} \ x \in [0,l], \ t \in [0,T].$$

The conjugate gradient algorithm (CGA) is designed for the numerical solution of the identification problem. The proposed approach is based on weak solution theory for PDEs, Tikhonov regularization combined with the adjoint method. Computational results, obtained for noisy output data, are illustrated to show an efficiency and accuracy of the proposed approach, for typical classes of source functions.

References:

[**1**] L. Fryba, Vibrations of the Solids and Structures under Moving Loads, Thomas Telford Publishing House, 1999.

[**2**] M. Yamamoto, Determination of forces in vibrations of beams and plates by point wise and line observations, J. Inv. Ill-Posed Problems, Vol.4, No.5, pp.437-457 1996.

This talk is based on the paper:

[1] Baysal, O., Hasanov, A., Kawano, A. **(2022)** Reconstruction of the spatial component in the source term of a vibrating elastic plate from boundary observation. *Appl. Math. Model.* 103, 409–420. `https://www.sciencedirect.com/science/article/pii/S0307904X21005175`

An inverse problem for a class of canonical systems with no indivisible intervals

Masatoshi Suzuki

Tokyo Institute of Technology

A Hamiltonian is a 2-by-2 positive semidefinite real symmetric matrix-valued function defined on an interval whose components are locally integrable. A canonical system is a first-order system of linear differential equations parametrized by complex numbers associated with a given Hamiltonian. The solution of a canonical system gives an entire function of the Hermite–Biehler class. In this talk, we solve the inverse problem for which recovers a Hamiltonian from a given function E in the Hermite–Biehler class under some special assumptions on E. The method of the solution is similar to the solution of the inverse problem for strings given by M. G. Krein, but is different. We will also explain the difference.

This talk is based on the papers:

[1] Suzuki, M. **(2020)** An inverse problem for a class of canonical systems having Hamiltonians of determinant one, *J. Funct. Anal.* 279, No. 12, 108699. `https://www.sciencedirect.com/science/article/pii/S0022123620302421`
[2] Suzuki, M. **(2020)** Chains of reproducing kernel Hilbert spaces generated by unimodular functions. `https://arxiv.org/abs/2012.11121`
[3] Suzuki, M. **(2022)** An inverse problem for a class of lacunary canonical systems with diagonal Hamiltonian, *Tohoku Math. J.* 74, No. 4. `https://arxiv.org/abs/1907.07838`

The recording of the talk can be found at `https://www.youtube.com/watch?v=kJdMjUZ562E`

Inverse problems for discrete operators

Vjacheslav A. Yurko

Saratov State University

We give a short review of results on inverse spectral problems for wide classes of discrete operators. We start with the simplest class of Jacobi operators. Then we will pay attention on other more complicated classes of discrete operators. We will use a unified approach for studying different classes of discrete operators.

This talk is based on the papers:

[1] Yurko, V. A. **(1996)** An inverse problem for operators of a triangular structure. *Results Math.* 30, 346–373. https://link.springer.com/article/10.1007/BF03322200
[2] Yurko, V. A. **(1995)** On higher-order difference operators. *J. Differ. Equ. Appl.*, 1:4, 347-352, doi: 10.1080/10236199508808033 https://www.tandfonline.com/doi/abs/10.1080/10236199508808033
[3] Yurko V. A. **1995** On integration of nonlinear dynamical systems by the inverse problem method. *Matem. Zametki*, 57, no.6, 945–949 (Russian); English transl. in *Math Notes*, 57, no.6, 672–675. https://link.springer.com/article/10.1007/BF02304569

The recording of the talk can be found at https://www.youtube.com/watch?v=3L0CBq3BfRI

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