Seminars on Inverse Problems

Theory and Applications

Book of Abstracts Vol. 1

March 8 – June 7, 2022

Foreword

Dear friends,

Since March 8, 2022, we have been getting together with people who feel enthusiastic about inverse problems on a fortnightly basis and we have had the opportunity to listen to talks and presentations by established mathematicians. In this short period of time, we were honored to host seven speakers from six different countries including Bosnia&Herzegovina, Denmark, Russia, Sweden, Türkiye, and the United States of America. Our titles varied from *Inverse spectral problems for Dirac operators with constant delay: uniqueness, characterization, uniform stability* to *The mathematics of "hearing the shape of a drum".*

In this book, you will find the abstracts of the talks, as well as the articles that these talks are based on, and the links to the recordings of the talks. By compiling this small book, we wanted to promote our speakers once again, and by doing that we wanted to thank them all one more time.

Our aim, when we decided to schedule this seminar series was, to foster connections among researchers working on various types of inverse problems and their applications. Hosting these seminars has given us the opportunity to catch up with old friends as well as meet some new incredible intellectuals, an opportunity we are enormously grateful for.

From June 7th, we are taking a break until the upcoming semester begins. We have already started getting in touch with new highly-distinguished speakers to set up our future talks. After recharging in the summer season we are looking forward to seeing you again in the fall.

We really appreciate your contribution, hard work, and time.

Please stay tuned and to be continued.

Co-organizers (listed in alphbetical order)

F. Ayça Çetinkaya[1], Vladimir Vladičić[2], Biljana Vojvodić[3]

Mersin University[1], University of East Sarajevo[2], University of Banja Luka[3]

Contents

Foreword **ii**

Abstracts **1**

Inverse spectral problems for Dirac operators with constant delay: uniqueness, characterization, uniform stability (*Sergey Buterin, Nebojša Djurić*) 1

A complex analytic approach to inverse spectral problems (*Burak Hatinoğlu*) 2

Inverse problem for a differential operator on a star-shaped graph with nonlocal matching condition (*Natalia Bondarenko*) 3

IR Tools: Iterative Regularization for Inverse Problems (*Per Christian Hansen*) 4

Inverse problem for the Sturm–Liouville operators with frozen argument (*Maria Kuznetsova*) 5

On the numerical solutions of some inverse source problems backward in time (*Ali Uğur Sazaklıoğlu*) 6

The mathematics of "hearing the shape of a drum" (*Julie Rowlett*) 7

Author Index **8**

Abstracts

March 8, 2022

Inverse spectral problems for Dirac operators with constant delay: uniqueness, characterization, uniform stability

Sergey Buterin[1], Nebojša Djurić[2]

[1]Saratov State University, [2]University of Banja Luka

We initiate studying inverse spectral problems for Dirac-type functional-differential operators with constant delay. For simplicity, we restrict ourselves to the case when the delay parameter is not less than one half of the interval. For the considered case, however, we give answers to the full range of questions usually raised in the inverse spectral theory. Specifically, reconstruction of two complex L_2-potentials is studied from either complete spectra or subspectra of two boundary value problems with one common boundary condition. We give conditions on the subspectra that are necessary and sufficient for the unique determination of the potentials. Moreover, necessary and sufficient conditions for the solvability of both inverse problems are obtained. For the inverse problem involving the complete spectra, we establish also uniform stability in each ball of a finite radius. For this purpose, we use recent results on uniform stability of sine-type functions with asymptotically separated zeros.

This talk is based on the preprint: Buterin, S., Djurić, N. Inverse problems for Dirac operators with constant delay: uniqueness, characterization, uniform stability, `https://arxiv.org/abs/2204.08259`

The recording of the talk can be found at `https://www.youtube.com/watch?v=_BVD8azlm7g`

March 22, 2022

A complex analytic approach to inverse spectral problems

Burak Hatinoğlu

UC Santa Cruz

In this talk we will consider the Schrödinger operator on a finite interval with an L^1-potential. Borg's two spectra theorem says that the potential can be uniquely recovered from two spectra. By another classical result of Marchenko, the potential can be uniquely recovered from the spectral measure or Weyl m-function. After a brief review of inverse spectral theory of one dimensional Schrödinger operators, we will discuss the following mixed spectral problem as a complex analysis problem: Can one spectrum together with subsets of another spectrum and norming constants uniquely determine the potential?

This talk is based on the paper: Hatinoğlu, B.**(2021)** Mixed data in inverse spectral problems for the Schrödinger operators, *J. Spectr. Theory* 11, 281-322. doi: 10.4171/JST/341 `https://ems.press/journals/jst/articles/299091`

The recording of the talk can be found at `https://www.youtube.com/watch?v=ntvelbv7l2Q`

Inverse problem for a differential operator on a star-shaped graph with nonlocal matching condition

April 5, 2022

Natalia Bondarenko

Samara National Research University, Saratov State University

In this talk, we consider the Laplace operator on a star-shaped graph with nonlocal integral matching condition. This operator is adjoint to the functional-differential operator with frozen argument at the central vertex of the graph. We study the inverse problem that consists in the recovery of the integral condition coefficients from the eigenvalues. The uniqueness theorem, the spectrum characterization, and two reconstruction algorithms have been obtained for this inverse problem. We will discuss the connection of our results with the study of inverse problems for functional-differential operators with frozen argument and other related issues.

This talk is based on the preprint: Bondarenko, N.P. Inverse problem for a differential operator on a star-shaped graph with nonlocal matching condition, `https://arxiv.org/abs/2201.10461`

The recording of the talk can be found at `https://www.youtube.com/watch?v=TltMTO6tazw`

April 19, 2022

IR Tools: Iterative Regularization for Inverse Problems

Per Christian Hansen

Technical University of Denmark

The Matlab package IR Tools provides implementations of a range of iterative solvers for linear inverse problems, and a set of large-scale test problems in the form of discretizations of 2D linear inverse problems. We include iterative regularization methods where the regularization is due to the semi-convergence, and Tikhonov-type formulations where the regularization is due to a regularization term. In both cases, we can impose bound constraints on the solution.

We implemented the iterative methods in a flexible fashion that allows the problem's coefficient matrix to be available as a (sparse) matrix, a function handle, or an object. The basic call to all of the iterative methods requires only this matrix and the right-hand side. Our codes automatically set default parameters of the stopping rules, regularization parameters, etc.; with an optional input structure, the user has full control of any of these algorithm parameters. The test problems represent realistic large-scale problems found in image reconstruction and several other applications. These new test problems replace the small and outdated test problems from 1994 in Regularization Tools. The basic call to all of the test problem generators produces a matrix, a right-hand side and the corresponding exact solution. Similar to the iterative methods, the user can use an optional input structure to control specific features of the test problem.

This talk is based on the paper: Gazzola, S., Hansen, P.C., Nagy, J.G. **(2019)** IR Tools: a MATLAB package of iterative regularization methods and large-scale test problems. *Numer Algor* 81, 773–811. doi: 10.1007/s11075-018-0570-7 `https://link.springer.com/article/10.1007/s11075-018-0570-7`

The recording of the talk can be found at `https://www.youtube.com/watch?v=95rA_vz9UU4`

Inverse problem for the Sturm–Liouville operators with frozen argument

May 3, 2022

Maria Kuznetsova

Saratov State University

The talk is devoted to recovering the Sturm–Liouville operator with frozen argument from its spectrum. Unique solvability of this inverse problem depends on the position of frozen argument and the boundary conditions. We compare different approaches to the inverse problem and the corresponding results in two cases of rational and irrational frozen argument. Further, we suggest a new unified approach to operators with frozen argument, which is effective in the both cases. Applying it, we obtain new-type asymptotic formulae completely characterizing the class of sequences that can be the spectra of the considered operators.

This talk is based on the paper: Kuznetsova, M. **(2022)** Necessary and sufficient conditions for the spectra of the Sturm–Liouville operators with frozen argument, *Applied Mathematics Letters* 131, article 108035. `https://www.sciencedirect.com/science/article/abs/pii/S0893965922000738`

The recording of the talk can be found at `https://www.youtube.com/watch?v=WGDXYHt4EvU`

May 17, 2022

On the numerical solutions of some inverse source problems backward in time

Ali Uğur Sazaklıoğlu

University of Turkish Aeronautical Association

In this talk, two inverse source problems for a one-dimensional, and for a multidimensional linear parabolic equations backward in time will be considered. In each of these problems we recover a spacewise dependent source term from an overdetermination. Note that if the conditions with respect to time are given at $t \neq 0$, then the problems become some inverse problems of simultaneous recovery of the source and the initial condition. For the analysis of these problems some stability estimates for their solutions will be presented in some Banach spaces. Furthermore, for the numerical solution of the problems some explicit finite difference schemes will be presented along with some stability estimates for their solutions. Finally, the proposed numerical methods will be performed on some test problems to acquire their initial conditions, sources, and solutions, simultaneously. Moreover, for showing the efficiency of the proposed methods a numerical analysis is given.

This talk is based on the paper: Sazaklıoğlu, A. U. **(2022)** On the numerical solutions of some identification problems for one- and multidimensional parabolic equations backward in time. *Applied Numerical Mathematics* `https://doi.org/10.1016/j.apnum.2022.05.015`

The recording of the talk can be found at `https://www.youtube.com/watch?v=0YFl-VGKXZs`

The mathematics of "hearing the shape of a drum"

June 7, 2022

Julie Rowlett

Chalmers University of Technology

Have you heard the question, "Can one hear the shape of a drum?" Do you know the answer? In 1966, M. Kac's article of the same title popularized the inverse isospectral problem for planar domains. Twenty-six years later, Gordon, Webb, and Wolpert demonstrated the answer, but many naturally related problems remain open today. We will discuss old and new results inspired by "hearing the shape of a drum."

This talk is based on the papers:

Lu, Z., Rowlett, J. **(2015)** The sound of symmetry. *Amer. Math. Monthly* 122 (9), 815–835.https://www.tandfonline.com/doi/abs/10.4169/amer.math.monthly.122.9.815

Lu, Z., Rowlett, J. **(2016)** One can hear the corners of a drum. *Bull. London Math. Soc.*. https://doi.org/10.1112/blms/bdv094

Hezari, H., Lu, Z., Rowlett, J. **(2017)** The Neumann Isospectral Problem for Trapezoids. *Ann. Henri Poincaré* 18 , 3759–3792. https://link.springer.com/article/10.1007/s00023-017-0617-7

Aldana, C.L., Rowlett, J. **(2018)** A Polyakov Formula for Sectors. *J Geom Anal* 28, 1773–1839. https://doi.org/10.1007/s12220-017-9888-y

Nursultanov, M., Rowlett, J., Sher, D. **(2019)** How to Hear the Corners of a Drum. In: de Gier, J., Praeger, C., Tao, T. (eds) 2017 MATRIX Annals. MATRIX Book Series, vol 2. Springer, Cham., https://doi.org/10.1007/978-3-030-04161-8_18

Nursultanov, M., Rowlett, J., Sher, D. **(2019)** The heat kernel on curvilinear polygonal domains in surfaces. Preprint under review. https://arxiv.org/abs/1905.00259

Hezari, H., Lu, Z., Rowlett, J. **(2021)** The Dirichlet isospectral problem for trapezoids. *J. Math. Phys.* 62, 05151. https://aip.scitation.org/doi/pdf/10.1063/5.0036384

Rowlett, J., Blom, M., Nordell, H., Thim, O., Vahnberg, J. **(2021)** Crystallographic groups, strictly tessellating polytopes, and analytic eigenfunctions. *American Math. Monthly* 128 (5) 387–406. https://www.tandfonline.com/doi/full/10.1080/00029890.2021.1890498

Nilsson, E., Rowlett, J., Rydell, F. The isospectral problem for flat tori from three perspectives, to appear in *Bulletin of the American Mathematical Society.* https://arxiv.org/abs/2110.09457

The recording of the talk can be found at https://www.youtube.com/watch?v=-cmhZ81Pi9A

Author Index

Bondarenko
Natalia, 3
Buterin
Sergey, 1

Djuric
Nebojsa, 1

Hansen
Per Christian, 4

Hatinoğlu
Burak, 2

Kuznetsova
Maria, 5

Rowlett
Julie, 7

Sazaklıoğlu
Ali Uğur, 6